MATHEMATICAL LOGIC HOMEWORK 3

Due Monday, February 26.

Problem 1. *Let $\mathcal{L} = \{s\}$, where s is a unary function symbol. Let T be the $\mathcal{L}$- theory that asserts that s is a bijection with no cycles (i.e., $s(n)(x) \neq x$ for $n = 1, 2, ...$). For which cardinals κ is T κ-categorical? Is T complete? Prove your answer.*

Problem 2. *Let $\mathcal{L} = \{E\}$, where E is a binary function symbol. Let $\mathcal{K}$ be the class of all models satisfying that E is an equivalence relation with infinitely many equivalence classes, all of which are infinite.*

(a) Show that $\mathcal{K}$ is an elementary class i.e. there is a theory T, such that $\mathfrak{A} \models T$ iff $\mathfrak{A} \in \mathcal{K}$.

(b) For which cardinals κ is T κ-categorical? Is T complete? Prove your answer.

Problem 3. *Let $\mathcal{L} = \{<, 0, 1, +, \cdot, S\}$ and let $\mathfrak{A} = (\mathbb{N}, <, 0, 1, +, \cdot, S)$ be the standard model of arithmetic (here $S(n) := n + 1$ is the successor function). Let T be the theory of $\mathfrak{A}$. Show that every model $\mathfrak{B}$ of T is an end-extension of $\mathcal{A}$, i.e. there is an embedding $f : \mathbb{N} \to B$, such that for all $b_1, b_2 \in B$, if $b_1 \in \mathrm{ran}(f)$ and $b_2 \notin \mathrm{ran}(f)$, then $b_1 <^{\mathfrak{B}} b_2$. (B is the universe of $\mathfrak{B}$; see Definition 1.13 for the definition of an embedding.)*

Problem 4. *Let $\mathcal{L} = \{<, 0, 1, +, \cdot, S\}$ and $\mathfrak{B}$ be a non standard model of the theory of $(\mathbb{N}, <, 0, 1, +, \cdot, S)$. I.e. there is an element b in the universe of $\mathfrak{B}$ such that for all n, $\mathfrak{B} \models S^n(0) < b$. We call such b's infinite. (Recall that we proved the existence of such models.) Show that if b is an infinite element, then there are infintely many infinite elements less than it.*

Problem 5. *Let κ be an uncountable cardinal. Show that DLO (the theory of dense linear order with no end points) is not κ-categorical.*